Enrollment No: ____________________ Exam Seat No: ____________________

C.U.SHAH UNIVERSITY

Winter Examination-2021

Subject Name : Mathematical Physics and Classical Mechanics
Subject Code : 4SC05MPC1 **Branch: B.Sc. (Physics)**
Semester: 5 **Date: 16/12/2021** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

1. (1) Use of Programmable calculator & any other electronic instrument is prohibited.
2. (2) Instructions written on main answer book are strictly to be obeyed.
3. (3) Draw neat diagrams and figures (if necessary) at right places.
4. (4) Assume suitable data if needed.

Q-1		**Attempt the following questions:**	**(14)**
	a)	Write Dirichlet Conditions.	(01)
	b)	Write the formula of half range Fourier coefficients.	(01)
	c)	What are the applications of the Fourier series in the field of physical sciences?	(01)
	d)	What kind of Fourier series' are obtained for even and odd functions?	(01)
	e)	Write the value / formula of a_0 , a_n and b_n in the interval of $(-\ell, \ell)$ for the extended intervals.	(01)
	f)	Γ (m+1) = ……..Γm = …….!	(01)
	g)	Γ (1/2) = ………	(01)
	h)	What is constraint and constrained forces?	(01)
	i)	Name different types of constraints.	(01)
	j)	Define:Generalized coordinates.	(01)
	k)	What is virtual displacement and virtual work?	(01)
	l)	Define: Degree of Freedom.	(01)
	m)	Define: Variational principle.	(01)
	n)	What are the ignorable and cyclic coordinates?	(01)

Attempt any four questions from Q-2 to Q-8

Q-2		**Attempt all questions**	**(14)**
	(A)	Prove: $\int_{-\infty}^{\infty} e^{-k^2x^2}\, dx = \sqrt{\pi}/k$	(04)
	(B)	Prove: $\int_0^{\infty} x^4\, e^{-x^4} dx = \frac{1}{16}\Gamma\left(\frac{1}{4}\right)$	(05)
	(C)	Prove: $\int_0^1 x^m \left(\log\frac{1}{x}\right)^n dx = \frac{n!}{(m+1)^{n+1}}$	(05)

Q-3		**Attempt all questions**	**(14)**
	(A)	Prove: $\beta(m,n) = \Gamma m.\Gamma n / \Gamma(m+n)$	(05)
	(B)	Prove: $\sqrt{\pi} = \Gamma\frac{1}{2}$	(03)
	(C)	Prove: $\Gamma n \Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2n-1}}\Gamma 2n$	(06)
Q-4		**Attempt all questions**	**(14)**
	(A)	Define Fourier series giving its general formula and Fourier coefficients. Obtain cosine and sine series.	(06)
	(B)	Obtain Fourier series of $f(x) = x^2$ in the interval $(-\pi, \pi)$ and prove: (1) $\frac{1}{1}+\frac{1}{2^2}+\frac{1}{3^3}+\cdots=\frac{\pi^2}{6}$ (2) $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{8}$ (3) $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots=\frac{\pi^2}{12}$	(08)
Q-5		**Attempt all questions**	**(14)**
	(A)	If the function $f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ 1 & 0 \le x \le \pi \end{cases}$ then prove that the Fourier series is $f(x) = \frac{1}{2}+\frac{2}{\pi}\left[\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots\right]$	(06)
	(B)	Derive Lagrange's Equation of Motion for the Non Conservative System.	(08)
Q-6		**Attempt all questions**	**(14)**
	(A)	Give the only final Lagrangian formula for the Simple, Compound, Double and Spherical pendulums. Derive any one of them.	(07)
	(B)	Derive formula for the velocity dependent potential of EM field.	(07)
Q-7		**Attempt all questions**	**(14)**
	(A)	Derive Hamilton's principle from Newton's equation.	(05)
	(B)	Discuss and derive necessary formula for the series and parallel L-C-R circuit using Lagrangian formulation and Rayleigh's dissipation function.	(09)
Q-8		**Attempt all questions**	**(14)**
	(A)	Obtain Hamilton's equation of motion and prove that H = K.E.+P.E.	(07)
	(B)	Obtain Hamilton of simple pendulum with moving support and derive the formula for the simple pendulum from Hamilton's equation.	(07)